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Quantum Statistics in a Simple Model of Space-Time[1]

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A classification of particles in two classes is proposed in the framework of a model of space-time previously introduced. One of them is constituted by particles being represented by sets of sets of preparticles. The other kind is constituted by particles represented by cuts or discontinuities in a space-time. We prove here that particles of the first and second kind follow the Bose–Einstein and Fermi–Dirac statistics, respectively.

1. INTRODUCTION

Before we give a summary of our model of space-time (in the next section), let us mention briefly the main trends in the development of foundational theories of this concept.

A first classification could be done according to the scale of phenomena to which these theories refer, i.e., in macroscopic and microscopic foundational theories of space-time. A typical macroscopic one is the Basri theory of space-time (Basri, 1966). Examples of microscopic theories are the Finkelstein theory (Finkelstein, 1969, 1972a, 1972b, 1974, 1982; Finkelstein et al., 1974), Penrose's theory (Penrose, 1967, 1968, 1975), Borneas's theory (Borneas, 1976, 1980, 1981), and Sachs's theory of space-time (Sachs, 1972, 1980, 1981).

Another criterion of classification could be the relational character of these theories. We consider a foundational theory of either space or space-time as relational in a strong sense when some kind of material objects and order relations between them enter in the theory at the level of primitive

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concepts, and when the disappearance of material objects entails the disappearance of space-time. When only this last condition is fulfilled, we consider the theory in question to be relational in a weak sense. Examples of relational macroscopic theories in a strong sense are those of Basri (1966), and Bunge and García-Máynez (1976). An example of a relational microscopic theory of space-time in a strong sense is that of Finkelstein. In the last version of this theory a quantum topology is elaborated in terms of some kind of material objects and causal relations between them (Finkelstein, 1982). The Penrose and Borneas theories, already mentioned, are examples of relational (in a weak sense) microscopic theories of space-time.

Another classification of space-time theories could be done according to the discreteness or continuity of the space-time. The relevance of this disjunctive has been stressed recently by several authors (see, e.g., Feynmann, 1982; Finkelstein, 1982).

A fourth criterion of classification could be the kind of logic underlying the theory under consideration. The majority of space-time theories have been elaborated assuming, either implicitly or explicitly, classical logic. All the theories mentioned above, except Finkelstein's, are classical-logic space-time theories.

A last criterion could be a "filogenetic" one, in the sense of the universally accepted physical theories, from which the considered foundational space-time theories emerge directly, by using the main concepts and the mathematical apparatus of such accepted theories. Needless to say, all foundational space-time theories are inspired by relativity. Yet, apart from this indirect relationship, there are space-time theories which use the fundamental concepts and the mathematical apparatus of general relativity. In this sense, such theories emerge directly from general relativity. Examples of those theories are the Basri, Borneas, and Sachs theories. The Penrose theory is related directly to both quantum theory and relativity; in fact, this theory is an attempt to conciliate them (Penrose, 1975).

The theory treated in this paper (see Section 2) is a microscopic, relational in a weak sense, discrete, and classical-logic theory of space-time. Furthermore, this theory is indirectly related to relativity and quantum theory in the sense mentioned above, although it has been elaborated trying to reproduce some main features of these two universally accepted theories.

Within the framework of this theory of space-time we will analyze in the present paper the problem of quantum statistics.

2. A SET-THEORETICAL THEORY OF SPACE-TIME

Here we are concerned with a model of space-time already developed in previous papers (García-Sucre, 1975, 1978a, 1978b, 1979, 1981). In the elaboration of this model we have started from the two primitive concepts of preparticles and the membership relation of set theory. We have considered preparticles as the most basic components of any physical system (García-Sucre, 1975).

Let $B \equiv \{\alpha_i / i \in I\}$, where I is a finite set of labels, B the set whose members are all the preparticles in the universe, which we assume to be finite in number. We have given arguments in previous papers in favor of representing particles as subsets of the power set $P(B)$ of B (García-Sucre, 1975, 1978a, 1978b, 1979). Let the set

$$p_i = \{a^i(x) | x \in X \text{ and } a^i(x) \in P(B)\} \tag{1}$$

where X is a finite set of labels, represents a particle in our model; the members $a^i(x)$ of p_i being ordered by the relation of proper inclusion $\subset$. If the set p_i representing a particle can be completely ordered by the relation $\subset$, then we say that p_i represents an evolving particle. Let us point out here that we will use the same symbol to denote a physical entity (e.g., particles, fields, etc.) and the set representing it in our model. On the other hand, we call those particles represented by partly ordered sets nonevolving particles (García-Sucre, 1975).

We call α state of p_i any set

$$s^i(x) = a^i(x) - \cup\, a^i(x'), \qquad x' \in X'(x) \tag{2}$$

where $a^i(x)$, $a^i(x') \in p_i$, $X'(x)$ being such that for every $x' \in X'(x)$ the relation $a^i(x) \not\subseteq a^i(x')$ holds. We denote $\sum(p_i)$ the set of all the α states of p_i ordered according to the following rule: given two α states $s^i(x)$, $s^i(y) \in \sum(p_i)$, then $s^i(x)$ precedes $s^i(y)$ if $a^i(x) \subset a^i(y)$, where $a^i(x)$ and $a^i(y)$ are related, respectively, to $s^i(x)$ and $s^i(y)$ through equation (2).

Let us illustrate the above definitions in the following graphic representation of particles. Imagine that a page like this one is divided in a very large but finite number of very small regions covering it completely. Assume that each small region stands for a preparticle. We represent a member $a^i(x)$ of p_i, equation (1), by marking with a pencil on the page all those small regions representing preparticles belonging to $a^i(x)$. For the sake of simplicity let us assume that each α state of p_i is a set to which only one preparticle belongs. Therefore, only one pencil mark on the page of our example will correspond to each α state of p_i. A set p_i fulfilling this condition is for instance

$$p_i = \{a^i(x_1), \ldots, a^i(x_n)\}$$

where $a^i(x_1) = \{\alpha_1\}$, $a^i(x_2) = \{\alpha_1, \alpha_2\}, \ldots, a^i(x_n) = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. Note that p_i is an evolving particle since

$$a^i(x_1) \subset a^i(x_2) \subset \cdots \subset a^i(x_n)$$

Furthermore the α states of p_i are given by [see equation (2)] $s^i(x_1) = a^i(x_1) = \{\alpha_1\}$, $s^i(x_2) = a^i(x_2) - a^i(x_1) = \{\alpha_2\}, \ldots, s^i(x_n) = a^i(x_n) - a^i(x_1) \cup \cdots \cup a^i(x_{n-1}) = \{\alpha_n\}$. The set $\sum(p_i)$ of α states of p_i is the completely ordered set $(s^i(x_1), s^i(x_2), \ldots, s^i(x_n))$. Assume that these states are distributed in our graphic representation along a continuous curve from a point **a** to a point **b** and they are ordered in the same way as the members of the set $\sum(p_i)$. In such a case the members $a^i(x)$s of p_i will appear in the page as all the initials determined on the curve *ab* by the successive pencil marks in the curves (the initial in *ab* determined by the pencil mark s of *ab* is the set of all the pencil marks of *ab* that precede s). Note that, as expected, all these initials can be ordered by the relation $\subset$. Briefly, the α states of p_i are graphically represented by the pencil marks giving rise to the curve *ab*, and the members of p_i are represented by the initials determined in *ab* by such pencil marks. We can endow the curve *ab* with an arrowhead to represent graphically the ordering of both the members and α states of p_i. If the particle under consideration is represented by a partly ordered set p_j, we can always express p_j as the union of sets $p_j^1, p_j^2, \ldots, p_j^k$, each of them being completely ordered in the same way as the set p_i above. Such subsets $p_j^1, \ldots, p_j^k$ of p_j are called branches of p_j. These branches may be such that they can be represented in our example above as continuous curves, each endowed with an arrowhead. In this case the nonevolving particle p_i will appear in our graphic representation either as a ramified graph, each branch endowed with an arrowhead, or a collection of curves, crossing or not to each other.

Note that the continuous curves with arrowheads representing evolving particles, or the more complicated ramified graph with continuous branches of our pictorial representation above, only serve to give a graphic and easily interpretable representation of orderings which, on the other hand, are completely determined by the way in which the members of the sets representing particles are included in each other [see equations (1) and (2)]. The pencil marks standing for the α states in our graphic representation of particles could appear sparsely on the page, yet the ordering of these marks could be still determined by looking at the set representing the particle under consideration. In this connection, note that permuting the positions of the small page regions in our example above we get different pages, all of them being equally legitimate for the graphic representation of particles. Still the same collection of particles would appear differently in each of these pages.

The richness with which we can specify orderings for members of sets by using the relation $\subset$ cannot be attained by ordering rules based on the proximity criterion of the positions of points on a page. However, we use such pictorial representations because they allow clear and intuitive graphic representations of physical systems.

We represent physical systems in our model as sets of sets representing particles (García-Sucre, 1975). Let S_c be a physical system with all its particles appearing on the page of our example as continuous curves, each one endowed with an arrowhead, and crossing to each other frequently. The structure of S_c is characterized by the way in which the α states of the particles belonging to S_c are ordered, and also by the crossings occurring between these particles. In more precise terms, we define a point of crossing of S_c as an ordered pair $(s^i(x);\ \prod_x^i(S_c))$, where $s^i(x)$ is an α state of a particle $p_i \in S_c$, and $\prod_x^i(S_c)$ is the set of all the particles belonging to S_c having α states which yield a nonempty intersection with $s^i(x)$ (García-Sucre, 1978a). We call $s^i(x)$ and $\prod_x^i(S_c)$ the center and the $\prod$ set of the point of crossing $(s^i(x);\ \prod_x^i(S_c))$. The set of all the points of crossing of S_c, which we denote as $\sum\sum(S_c)$, characterizes the structure of S_c. In our representation above the physical system S_c will appear as network and the points of crossing of S_c as the knots of this network.

We will say that two points of crossing $(s^i(x);\ \prod_x^i(S))$ and $(s^j(y);\ \prod_y^i(S))$ have the same structure, or are similar to each other (García-Sucre, 1979), when there exist two pages of the kind discussed above such that the representations in these two pages of these two points of crossing are identical. We will denote $\sim$ as the similarity relation between points of crossing.

We have given in a previous paper arguments in favor of representing the field produced by a physical system S as the quotient set $\sum\sum(S)/\sim$, each point of the field being represented as a member of $\sum\sum(S)/\sim$, i.e., as an equivalent class of points of crossing with respect to the relation $\sim$ of a similar structure (García-Sucre, 1978a, 1979). The main features of this choice are the following (García-Sucre, 1978a, 1979):

(i) Every point of the field is completely characterized with respect to the remaining points of the same field since from the definition of quotient set it follows that the members of $\sum\sum(S)/\sim$ all have different structures.

(ii) A topology can be ascribed to the field produced by any physical system. In this sense, a field can be visualized as a collection of points (equivalence classes) which are connected by particles. In more precise terms, given two points $x, x' \in \sum\sum(S)/\sim$ we say that they are connected by a particle p if there exist two points of crossing $\sigma \in x$ and $\sigma' \in x'$ such that p crosses over the centers of both σ and σ'. The way in which the points of a field are connected to each other will determine the topology of the field.

(iii) A clear definition can be given of detector physical systems which make room to the property of particles according to which they may appear sometimes as localized objects, and some other times as extended physical entities (García-Sucre, 1979, Section 3).

(iv) Space-time is defined in our model as a global field produced by a collection of physical systems $S_1, S_2, \ldots, S_n$. If this collection is such that the physical system $S \equiv S_1 \cup S_2 \cup \cdots \cup S_n$ has an extremely large number of particles, then the corresponding space-time $\mathrm{ST} = \sum \sum (S)/\sim$ can be expected to have a very large number of points, each pair of these points being connected by a large number of particles of S. Precisely, it is this richness in the connections between points that allows the definition of very many reference frames inside the space-time ST. In fact, a reference frame is obtained in our model when we select a subset of points of ST and a subset of connections between these points (García-Sucre, 1978a, 1978b, 1979).

Since in previous papers we have given a description for photons compatible with the invariance of the velocity of light (García-Sucre, 1978b, 1979) and with the "particlelike" and "wavelike" properties that particles may manifest on different physical situations (García-Sucre, 1979), we have tried to answer in the present paper the question whether the quantum statistics can be properly described in the framework of the same model.

3. REFERENCE FRAMES AND THE STATE OF A SYSTEM

The trajectory T_p^f of a particle p in the space-time $\mathrm{ST}(S)$ is defined as the set of all those points of ST over which the particle p passes; these points being ordered according to the ordering of the α states of p in the set $\sum(p)$. More precisely, we say that a particle p passes over the point x of ST when there exists at least one point of crossing $\left(s^i(x); \prod_x^i (S)\right) \in x$ such that at least one α state s of p fulfills $s \cap s^i(x) \neq \phi$. Equivalently we say that the α state s of p intersects the point x of ST. The ordering of the points of T_p^f is induced by the ordering of the corresponding α states of p. For instance, if each α state of p intersects only one point of ST, and p is an evolving particle, then the trajectory T_p^f will be a completely ordered set.

Making use of the concept of trajectory of a particle in a given space-time, we will arrive to define reference frames and then to ascribe unambiguously space and time coordinates to the points of a region of a space-time with respect to a given reference frame. Our definition of a reference frame R_{ST} in a space-time $\mathrm{ST}(S) = \sum \sum (S)/\sim$ consists in specifying, firstly, which points of ST are covered by R_{ST}, and secondly, which particles of S are selected in order that the points of R_{ST} are connected in a so simple and coordinated way that space and time coordinates can be ascribed to the points of R_{ST} in an unambiguous and simple way. For instance, we can have reference frames covering the same points of ST, but differing in the way in which those points are connected. A case in which this occurs is when the considered reference frames cover completely ST. In the above sense, we can consider a reference frame defined in a space-time to be inside this space-time.

More precisely, we define a reference frame R_{ST} as a triad $(R^0_{ST}; \tau; \varepsilon)$, where $R^0_{ST} \subseteq$ ST, and τ and ε are sets of particles selected out of S so that the conditions discussed below are fulfilled (García-Sucre, 1979).

Any set τ must fulfill that the trajectories T_p, $p \in \tau$, are completely ordered sets, all having the same number of points, without crossing each other, and any point of R^0_{ST} appears in only one of such trajectories.

We use the set τ of a reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$ to ascribe a time coordinate to every point of R^0_{ST}. Given that every T_p is a completely ordered set we can choose a point $x \in T_p$ as an origin and then to ascribe time coordinates to all the remaining points of T_p by counting how many points are found between the point $y \in T_p$ under consideration and the origin x. We assign the time coordinate zero to the origin x. The time coordinate of a point $y \in T_p$ such that $y > x$ (x precedes y) is given by the number of points of T^f_p which are found between x and y plus one. If $y < x$, then we consider the negative of the number of points between x and y minus one. Let us assume that the point $x \in T_p$ which we have chosen as origin is the nth member of T^f_p. We can proceed in the same way as above for all remaining trajectories T_p with $p \in \tau$. In particular, we chose as the origin in each of them its nth member. By doing so, we have arrived to ascribe a time coordinate to every point covered by R_{ST} and we have established similarity mappings between the trajectories T_p with $p \in \tau$, such that points having the same time coordinate are in correspondence with each other. The vertical lines in Figure 1, together with the circles over which they pass, stand for trajectories T_p, $p \in \tau$, associated to the reference frame represented in this figure.

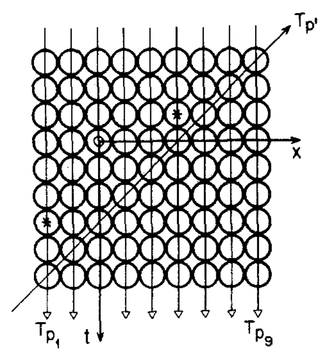

Fig. 1. A reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$. Circles stand for points of the space-time $ST = \sum \sum (S)/\sim$. The slanted arrow and the circles crossed over by this arrow stand for the trajectory of $p' \in \varepsilon$. Each vertical arrow and the circles over which it passes represent an evolving particle belonging to τ. The time and space separation between the starred points are 4 and 5 units, respectively. The order of immediate connection of any trajectory belonging to τ is either 2 or 1. The space and time axes associated to this reference was frame cross over the point ○ arbitrarily chosen as the origin.

On the other hand, the set ε of the reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$ serves to the assignment of the space coordinates to every point of R_{ST}. To the set ε belongs particles of S whose trajectories in ST are completely ordered sets. Furthermore, a particle belonging to ε intersects a particle belonging to τ in only one point or does not intersect it at all. The particles of τ can be ordered according to the ordering of the points of the trajectories of the particles belonging to ε in the following way. Given a particle $p' \in \varepsilon$ such that it intersects the particles p_i and p_j of τ at the points x and y, respectively, then p_i precedes p_j if x precedes y in the trajectory $T_{p'}$. If in such an ordering of the particles belonging to τ the particle p_i is either the immediate successor or the immediate antecedent of p_j we say that the trajectories T_{p_i} and T_{p_j} are immediately connected by the trajectory $T_{p'}$. We denote $\nu^\varepsilon_\tau(T_p)$ as the number of all the trajectories of particles belonging to τ which are immediately connected to T_p by trajectories of particles belonging to ε. For instance, the reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$ represented in Figure 1 is such that only one particle belongs to ε, namely, p'. Furthermore, the function ν^ε_τ can only take the values 1 or 2 in the domain of trajectories T_p of particles belonging to τ. More generally, in addition to the properties of the particles belonging to ε which we have announced above, those particles must fulfill the following properties. Let $\tilde{\tau}(p')$ be the subset of τ whose particles are intersected by a particle $p' \in \varepsilon$. Then, once the trajectories T_p with $p \in \tilde{\tau}(p')$ have been ordered by a particle $p' \in \varepsilon$ according to the rule given above, this ordering remains unaltered when any other particle belonging to ε is considered. Also we must have $U_{p' \in \varepsilon} \tilde{\tau}(p') = \tau$ and that $\nu^\varepsilon_\tau(T_p)$ is equal to either $2n$ or $2n-1$, where n is an integer, for any $p \in \tau$ (García-Sucre, 1979).

The space separation between two points $x \in T_{p_i}$ and $y \in T_{p_j}$ in the reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$, where $p_i, p_j \in \tau$, is given by one plus the number of trajectories T_ps, with $p \in \tau$, which are found in R_{ST} between T_{p_i} and T_{p_j}. In the example illustrated in Figure 1 the space separation between the two starred points is equal to five vertical trajectories. In the same way, the time separation between these two points is equal to four horizontal rows of points. Note that these horizontal rows are constructed according to the mapping between points having the same time coordinate which we have described above.

The values that $\nu^\varepsilon_\tau(T_p)$ takes for different trajectories T_ps, with $p \in \tau$, are related to the spatial dimensionality of the reference frame R_{ST} under consideration (García-Sucre, 1979). We have stated above that the function ν^ε_τ must be equal to either $2n$ or $2n-1$ in the domain of trajectories T_p with $p \in \tau$, entering in the reference frame $R_{ST} = (R^0_{ST}; \tau; \varepsilon)$. If all the trajectories T_p, $p \in \tau$, are such that $\nu^\varepsilon_\tau(T^f_p) = 2n$, we say that R_{ST} is n-dimensional without boundaries. On the other hand, if the function ν^ε_τ takes the value $2n$ for some trajectories T_ps, with $p \in \tau$, and $2n-1$ for some other trajectories T_p, we say that R_{ST} is n-dimensional with an $(n-1)$-dimensional boundary. For instance, the reference frame represented in Figure 1 is such that ν_τ^ε is either equal to 2 or 1. Therefore, this reference frame is 1-dimensional with a 0-dimensional boundary. In fact, once we choose a point, say, 0, as origin in the reference frame appearing in Figure 1, there exists only one spatial axis passing over that point. Also there is only one time axis passing through the point 0, namely, the vertical oriented line crossing 0 in Figure 1. Brief, the space-time diagram associated with the reference frame appearing in Figure 1 has one spatial and one time-bounded axis. It can be easily seen that if for every trajectory T_p with $p \in \tau$ the function ν_τ^ε were always equal to 2, then the spatial axis in Figure 1 would be unbounded and the time axis could be either bounded or unbounded.

In order to have an intuitive representation of a reference frame $R_{\mathrm{ST}} = (R_{\mathrm{ST}}^0; \tau; \varepsilon)$ for which $\nu_\tau^\varepsilon(T_p)$ is equal to either 4 or 3, we can imagine several figures of the same kind as Figure 1 and distribute them in parallel plans connected by particles belonging to ε. These particles will induce an ordering for such plans in the same way as the particle p' induces an ordering for the trajectories T_ps in Figure 1. In such a reference frame one could define two spatial axes and one time axis crossing over a point of ST arbitrarily chosen as origin in R_{ST}.

Recall that a preparticle α_i enters in a point $x \in \mathrm{ST}$ if α_i belongs to the union of the centers of the points of crossing belonging to x. In other words, all those points of ST in which preparticles belonging to the α states of p enter, belong to the trajectory T_p. Note that the same preparticle α_i may enter in several points of ST. This is so because the only condition for a point of crossing σ to belong to a point $x \in \mathrm{ST}$ is that σ have the same structure as any other $\sigma' \in x$, and the centers of point of crossing having different structures may yield nonempty intersections. Therefore, the condition according to which the trajectories T_p with $p \in \tau$ and $T_{p'}$ with $p' \in \varepsilon$ are all completely ordered sets, and that we have required in our definition of reference frame, is a very restrictive one. Yet, if we want to relax this restriction we can substitute in our definition of reference frame above the points of trajectories T_p by clusters of point of ST, each of such clusters being formed by all the points of ST in which enter preparticles of a given α state of a particle $p \in \tau$. Since each cluster corresponds in this way to an α state, it follows that if all the particles belonging to τ are evolving particles, then these particles will induce orderings for the clusters in the same way as they induced orderings for the points of ST, as mentioned before. For reference frames in which many space-time points enter in each of such clusters we consider these clusters as if they were points.

Given a particle *p*, the trajectory of this particle in the reference frame R_{ST} will be the set $T_p \cap R_{ST}^0$, ordered according to the time coordinates in R_{ST} (García-Sucre, 1978b). We denote this set as $T(p; R_{ST})$. This definition corresponds well to the way in which we proceed in practice when we observe the track of a particle in a given reference frame. In such observations one considers that those sectors of the particle track corresponding to smaller time coordinates, precede in the trajectory those sectors corresponding to larger time coordinates being ascribed according to the time axis of the reference frame where the particle is being observed.

As a last point of this section we will introduce the concept of *state of a particle in a given space-time.* We will need this concept in the next section where we analyze the quantum statistics in the framework of our model of space-time.

One property usually required to be fulfilled by the concept of state of a physical entity is that by specifying the state of a physical entity we give the maximum possible information about this physical entity. On the other hand, one crucial assumption of our model is that the points of space-time can be distinguished from each other because they have different structures (García-Sucre, 1978b, 1979, 1981). Then, we adopt the point of view that the most detailed physical information that can be given about a particle in a space-time is the list of points of crossing in which this particle enters, specifying the structure of each of these points of crossing. Let us denote $s(p; \mathrm{ST})$ the state s of a particle p in the space-time ST. Therefore, we define $s(p; \mathrm{ST})$ as the set of points of crossing belonging to the points of ST in which the particle p enters.

The above definition of state has the property that once the state of a particle p in a space-time ST is given then the trajectory of this particle p is specified in each reference frame defined inside the space-time ST.

Recall, in this sense, that our definition of trajectory recover the particular case of a classical trajectory, reducing to it when corresponding to well-defined velocities and positions (García-Sucre, 1979, Section 3). Also, by given $s(p; \mathrm{ST})$ we are specifying the way in which the particle p partially determines the space-time ST, which in our formalism can be seen as a global field (García-Sucre, 1979, Sections 1 and 2).

4. QUANTUM STATISTICS

In our model a space-time is completely characterized by its field points and the way these points are connected to each other. The trajectories of all the particles that we have considered until now have to do with field points, and the intersections of the α states of these particles with the centers of points of crossing entering in these field points. We will consider that all these particles [i.e., those defined by equation (1)] belong to one kind of particle. Arguments will be given below to represent bosons in our model by particles of this kind.

On the other hand, we assume that fermions can also be represented in our model by a second kind of particles whose nature is related to the way the field points of ST are connected to each other. We represent a particle of this kind as a cut or discontinuity in ST. Such cuts in the space-time ST are characterized by the topology of ST (García-Sucre, 1979). Such particles will appear in different R_{ST}s reference frames as the corresponding cut of ST appears in these reference frames. Recall in this concern that a reference frame R_{ST} is specified by a subset of points of ST and a subset of connections between these points.

We have the following properties for the particles of the first kind, i.e., those represented by sets of sets of preparticles:

(i) Given a particle p and the space-time ST, the state $s(p;\,\mathrm{ST})$ of p in ST uniquely determines the trajectory $T(p;\,R_{\mathrm{ST}})$ of p in each reference frame R_{ST}. To see this let us start from the set $s(p;\,\mathrm{ST})$ of points of crossing belonging to the points of ST in which the particle p enters. The specification of $s(p;\,\mathrm{ST})$ determines the points of ST for which there are α states of p that have a nonempty intersection with the center of one or several points of crossing entering in these points of ST. Therefore, $s(p;\,\mathrm{ST})$ determines the trajectory $T(p;\,R_{\mathrm{ST}})$ in any R_{ST}.

(ii) An arbitrary number of particles of the first kind can be found in the same state.

Two particles, p and p', have the same state in ST if $s(p;\,\mathrm{ST}) = s(p';\,\mathrm{ST})$. This will occur when these two sets have the same elements, i.e., for any given point of crossing in which p enters, p' also enters and vice versa. There is no restriction in our model in the number of particles, except that the number of preparticles is finite, though very large (García-Sucre, 1979). In our model, the total number of particles of the first kind is even larger, since if N is the total number of preparticles, then N^N is the total number of such particles, according to equation (1). Then, for a sufficiently large space-time ST it can occur that there exists a large number of particles of the first kind, all entering in the same points of crossing. All these particles will have the same state in ST.

(iii) Systems of particles of the first kind follows the Bose–Einstein statistics (B.E.).

According to the property (i) above, only one state corresponds to a particle of the first kind in ST. This state, $s(p;\,\mathrm{ST})$, corresponds in turn to only one trajectory, $T(p;\,R_{\mathrm{ST}})$, in each reference frame R_{ST} defined in ST. Whatever the method we use to ascribe energy values to trajectories, we assume that we are dealing with particles for which one can ascribe an energy value to each trajectory in each reference frame. In a previous paper we have shown that defining inertial reference frames as those for which the number of preparticles entering in each of its field points is the same, we can ascribe energy values to trajectories corresponding to well-defined velocities (García-Sucre, 1979, Section 3). We have also shown that these energy values are compatible with the relation $E=\hbar\omega$ between energy and angular frequencies. Let us then assume that we are ascribing energy values to trajectories according to the method described in this previous work.

For a system of particles of the first kind $S=\{p_1, p_2, \ldots, p_N\}$ we have that for each particle $p_i \in S$ there is only one state $s(p_1;\,\mathrm{ST})$ and only one trajectory $T(p_i, R_{\mathrm{ST}})$ in each reference frame R_{ST}. This implies that in our model, given a reference frame the same particle cannot be found in different states, and this immediately leads to the partition function

$$Z=\sum_{n_1, n_2, \ldots, n_l} \exp[-\beta(n_1\varepsilon_1+n_2\varepsilon_2+\cdots+n_l\varepsilon_l)] \tag{3}$$

where n_j is the number of particles of S in the state j, ε_j is the energy corresponding to this state j in the considered reference frame, and $\sum_{j=1}^{l} n_j = N$, where N is the total number of particles entering in the system S. In equation (3) we are assuming that each state j is stationary and thus to it corresponds only one energy value ε_j.

Note in equation (3) that in front of each term of the sum the factor $N!/n_1!n_2!\ldots n_l!$ does not appear; this factor would take into account the possible ways in which particles "can be put" into given single-particle states $j=1, l$. The inclusion of these factors in equation (3) leads, as it is well known, to the Maxwell-Boltzmann statistics (MB). Finally, it is well known that the partition function in equation (3) and the restriction $\sum_{j=1}^{l} n_j = N$ lead to the B.E. distribution (see, e.g., Reif, 1963).

Let us now consider the case of the second kind of particles. These particles are represented in our model by cuts or discontinuities in ST. Following the same point of view as at the end of Section 3, the state $s(p;\,\mathrm{ST})$ of a particle p of this kind in the space-time ST will be specified by the set of points of crossing entering in the points of ST which are found at the boundary of the considered cut in ST. Let us denote $T(p;\,R_{\mathrm{ST}})$ as the trajectory of p in R_{ST}: the set of points at the boundary of the cut ordered according to the time coordinates in R_{ST}. The above definition of the state of a particle of the second kind in a given space-time, leads immediately to the property according to which no more than one particle p of the second kind can be found in a state $s(p;\,\mathrm{ST})$. To see this, let us assume that two different particles, p and p', are in the same state, in a given ST, i.e., $s(p;\,\mathrm{ST})=s(p';\,\mathrm{ST})$. This implies that the set of points of crossing specifying the state of p and p' in ST is the same. Then the points of ST which are found at the boundary of the cuts in ST representing p and p', respectively, are also the same. But this set of points of ST characterizes completely one cut in ST since it specifies the region of R_{ST} where the cut is found (recall that every point of ST has a different structure from any other point of ST), as well as the topological properties of this cut, since having the points of crossing we also have the way in which the points of ST are connected to each other. Therefore, p and p' are represented by the same cut in ST and thus, according to our definition of a particle of the second kind, $p = p'$.

On the other hand, given p, ST, and R_{ST}, the trajectory $T(p; R_{\text{ST}})$ is uniquely determined. Therefore, the partition function for a system of particles of the second kind will be that given in equation (3) with the restrictions that no more than one particle can occupy a given state and that $\sum_{j=1}^{l} n_j = N$. It is well known that equation (3) and these two restrictions lead to the Fermi–Dirac distribution (F.D.).

In the framework of our model the basic difference between M.B. and both B.E. and F.D. statistics consists in that what is a particle in M.B. statistics corresponds in fact to a collection of particles in B.E. and F.D. Let us examine this question by giving an example. Consider a system of macroscopic bodies $M_1, M_2, \ldots, M_r$. Classically, each of these bodies can occupy different states. The way in which one of these bodies, say, M_i, can change of state between states of different energy is by interacting with other bodies. If we disregard action-at-a-distance forces, we know that the interaction between particles and thus also between macroscopic bodies, is mediated by the exchange of particles. Therefore, when we say that the macroscopic body M_i can occupy the states $1, \ldots, n$ corresponding to different energies, we are in fact saying that different collections of particles, each considered as a unity, occupy different states. Classically, we identify all these collections of particles with only one system of particles, the body M_i, and describe the situation by saying that M_i changes, passing in this way by different states. If we consistently identify in such a way the macroscopic bodies and we count them accordingly, as well as the states they can occupy, then the M.B. statistics will be the correct statistics to describe systems of such bodies, as it is in fact. This corresponds to include a factor $N!/n_1!n_2!\ldots n_l!$ for each term of the sum in equation (3).

On the other hand, if we describe a physical system in a detailed way, i.e., at the microscopic level, we have three options:

(i) A microscopic particle can occupy different states, behaving in this respect as a macroscopic body.

(ii) Microscopic particles fulfill option (i) plus the indistinguishability postulate.

(iii) A microscopic particle cannot occupy different states in a given reference frame. There is a one-to-one correspondence between microscopic particles and single-particle states.

Option (i) is excluded by the experimental results. Option (ii) plus the postulate of the symmetric or antisymmetric character of the wave functions is in agreement with the experimental results. In option (ii) the indistinguishability of particles compensates exactly the number of states in excess introduced by the hypothesis that a microscopic particle can occupy different states. In other words, while this last hypothesis leads to the introduction of factors $N!/n_1!n_2!\ldots n_l!$s in equation (3), the indistinguishability postulate removes them again.

Option (iii), suggested by our model, is also in agreement with experimental evidence, yet simpler than option (ii), since it does not include any postulate about the indistinguishability of particles.

5. CONCLUDING REMARKS

In spite of the fact that our model does not describe quantum statistics in a detailed way, since no correlate of the spin is given in the present paper, it describes general traits of particles which seem to be sufficient in the framework of our model to obtain the correct quantum distributions for the two kinds of particles that we have postulated here. The difference between these two kinds of particles suggests that a possible correlate in our model for the spin could be the connectivity of the space-time where the particle under consideration is described. This possibility will be studied elsewhere.

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